

Reply on the “Comment on ‘Loss-error compensation in quantum-state measurements’ ”

T. Kiss^a, U. Herzog^b, and U. Leonhardt^c

^a Research Laboratory for Crystal Physics, Hungarian Academy of Sciences, P.O. Box 132, H-1502 Budapest, Hungary

^b Arbeitsgruppe “Nichtklassische Strahlung”, Institut für Physik der Humboldt Universität zu Berlin, Rudower Chaussee 5, 12484 Berlin, Germany

^c Abteilung für Quantenphysik, Universität Ulm, 89069 Ulm, Germany

The authors of the preceding Comment [G. M. D’Ariano and C. Macchiavello Phys. Rev. A (preceding comment), quant-ph/9701009] tried to reestablish a 0.5 efficiency bound for loss compensation in optical homodyne tomography. In our reply we demonstrate that neither does such a rigorous bound exist nor is the bound required for ruling out the state reconstruction of an individual system [G. M. D’Ariano and H. P. Yuen, Phys. Rev. Lett. **76**, 2832 (1996)].

03.65.Bz, 42.50.Dv

There is little doubt that the compensation of detection losses is a numerically delicate procedure that is extremely sensitive to experimental inaccuracies. We have shown [1,2], however, that no clear efficiency bound exists beyond that loss compensation is impossible, in contrast to a statement in an earlier paper [3]. In the preceding Comment [4] D’Ariano and Macchiavello tried to reestablish a 0.5 bound for the overall efficiency η .

In this Reply we show that their analysis is incomplete and that still no in-principle bound exists. Furthermore we point out that the existence of such a compensation bound does not follow directly from the arguments given in Ref. [5] where the impossibility of measuring the state of an individual quantum system was proven.

What is the problem? We have shown [1] that a generalized Bernoulli transformation describes the influence of detection losses on the density matrix $\langle m|\hat{\rho}|n\rangle$ that is reconstructed from measured data obtained e.g. from optical homodyne tomography [6]. We have treated the loss process separately from the particular detection scheme. The inversion of the Bernoulli transformation [1] produces the unperturbed density matrix of the signal

$$\langle m|\hat{\rho}_{sig}|n\rangle = \lim_{j_M \rightarrow \infty} \sum_{j=0}^{j_M} B_j(\eta^{-1}) \langle m+j|\hat{\rho}_{meas}|n+j\rangle \quad (1)$$

with

$$B_j(\eta) = \eta^{(m+n)/2} (1-\eta)^j \left[\binom{m+j}{m} \binom{n+j}{n} \right]^{1/2}. \quad (2)$$

We have shown [1] that the convergence of the series (1) is not necessarily restricted to the range $0.5 < \eta \leq 1$, although the matrix elements $B_j(\eta^{-1})$ of the inverse transformation are divergent if $\eta < 0.5$ [7]. Furthermore [2]

we could employ multiple runs of compensation or the analytic continuation of the series (1). In this case no convergence bound on η does exist and hence loss compensation is possible *in principle*.

In practice, of course, experimental imperfections will affect the compensation of detection losses [8]. We have never claimed that loss compensation is easy [9], in fact, we have stressed [1] that for $\eta < 0.5$ other errors are amplified, e.g. the effect of any uncertainty in η itself. D’Ariano and Macchiavello [4] considered the influence of statistical errors for homodyne measurements. They assumed a finite number N of experiments, i.e. a finite statistical ensemble of individual quantum systems. In this case the reconstructed density matrix $\langle m|\hat{\rho}_{meas}(N)|n\rangle$ is an estimation of the matrix $\langle m|\hat{\rho}_{meas}|n\rangle$ with statistical error bars calculated according to Ref. [10]. However, as a fundamental axiom of quantum mechanics, the estimation must tend to the ensemble average when the ensemble size approaches infinity. Therefore, if we keep the cut-off j_M in the series (1) at an arbitrarily large, but fixed value and increase the number N of experimental runs, we must approach the correct result $\langle m|\hat{\rho}_{sig}|n\rangle$, with an arbitrarily small, fixed systematic error that depends only on the cut-off. D’Ariano and Macchiavello [4] did exactly the opposite. They fixed the size N of the statistical ensemble and increased the cut-off j_M , and found that the series diverges for $\eta < 0.5$. Our Fig 1. illustrates the influence of varying both j_M and N . We see clearly that the order of the limits $j_M \rightarrow \infty$ and $N \rightarrow \infty$ is important. We also see that quite a large number N of runs is required to produce faithful data for compensating a low efficiency η . D’Ariano and Macchiavello [4] discussed certainly an interesting aspect of loss compensation but, as we have seen, their analysis was incomplete.

Is the compensation bound [3] relevant for some more fundamental features of quantum mechanics than merely technical points of measurement technology? Does the 50% bound [3] rule out the state measurement of an individual system [5]? If the bound existed it clearly would, as was pointed out in Ref. [5]. From the conclusion, however, does not follow the premise. The impossibility of measuring the wave function does not imply that the 50% compensation bound exists.

The problem discussed in Ref. [5] is again a matter of performing limits in the right order. Suppose one taps a series of N probe beams from an individual light mode and performs a state reconstruction using the N

probes as a statistical ensemble. The effect of tapping, i.e. beam-splitting, is equivalent to detection losses [11] with an efficiency η scaling like N^{-1} . Therefore, when we attempt to reconstruct the quantum state of the individual light mode we should employ infinitely many probes, yet with infinitely poor efficiency. Not surprisingly, we cannot compensate the losses in this situation. A general 50% efficiency bound is much too much to be required for such a delicate matter.

In conclusion, compensation for low overall detection efficiency is numerically difficult. The value 0.5 for η plays clearly a crucial role [3] because at this value the matrix elements in the inverse Bernoulli transformation become unbounded [1]. Our analysis shows, however, that 0.5 is neither a rigorous bound for compensating losses in optical homodyne tomography, nor is this bound required for ruling out the state measurement of an individual system [5].

T.K. acknowledges the support by the National Research Fund of Hungary (OTKA) under Contract No. F017381 and Hungarian Academy of Sciences Grant No. 96/64-17 and by an Eötvös Fellowship. T.K. is grateful to Professor W. P. Schleich for his kind hospitality during his stay in Ulm. U.L. was supported by the Deutsche Forschungsgemeinschaft.

Ref. [12].

- [9] U. Leonhardt and H. Paul, J. Mod. Optics **41**, 1427 (1994).
- [10] U. Leonhardt, M. Munroe, T. Kiss, Th. Richter, and M. G. Raymer, Opt. Commun. **127**, 144 (1996).
- [11] U. Leonhardt and H. Paul, Phys. Rev. A **48**, 4598 (1993).
- [12] U. Leonhardt, *Measuring the Quantum State of Light*, (Cambridge University Press, Cambridge, 1997).

FIGURE CAPTION

Plot of the loss-compensated density-matrix element ρ_{00} with varying cut-off j_M and ensemble size N . We employed the same thermal state with $\bar{n} = 2$ as in the preceding Comment [4] and used an efficiency η of 0.48. We performed Monte-Carlo simulations to model a realistic experimental situation. First, we reconstructed the density matrix $\langle m | \hat{\rho}_{meas} | n \rangle$ from N runs of the computer experiment using the method developed in Ref. [10]. Then we performed the loss compensation (1) with varying cut-off j_M . We found that for a given j_M the reconstructed matrix elements ρ_{00} do approach the actual value of $\langle 0 | \hat{\rho}_{sig} | 0 \rangle = 0.33$ for increasing numbers N of runs, apart from a small systematic error. On the other hand, if we keep the number N of runs constant and increase the cut-off j_M the matrix element diverges [4], as can be seen from the behavior of ρ_{00} for $N = 10^3$. Thus, the order of the limits $j_M \rightarrow \infty$ and $N \rightarrow \infty$ is vital to the loss-compensation procedures [1,2].

-
- [1] T. Kiss, U. Herzog, and U. Leonhardt, Phys. Rev. A **52**, 2433 (1995).
 - [2] U. Herzog, Phys. Rev. A **53**, 1245 (1996).
 - [3] G. M. D'Ariano, U. Leonhardt, and H. Paul, Phys. Rev. A **52**, R1801 (1995).
 - [4] G. M. D'Ariano and C. Macchiavello, Phys. Rev. A (preceding comment), quant-ph/9701009.
 - [5] G. M. D'Ariano and H. P. Yuen, Phys. Rev. Lett. **76**, 2832 (1996).
 - [6] For experiments see for example D. T. Smithey, M. Beck, M. G. Raymer, and A. Faridani, Phys. Rev. Lett. **70**, 1244 (1993); M. Munroe, D. Boggavarapu, M. E. Anderson, and M. G. Raymer, Phys. Rev. A **52**, R924 (1995); S. Schiller, G. Breitenbach, S. F. Pereira, T. Müller, and J. Mlynek, Phys. Rev. Lett. **77**, 2933 (1996); G. Breitenbach, S. Schiller, and J. Mlynek, Nature **387**, 471 (1997). The theory is described in Chapter 5 of Ref. [12].
 - [7] It is interesting to note that the divergence occurs also for a classical particle-counting procedure [1]. For a quantum damping process a 50% loss marks the critical value where the Wigner functions of initial states become entirely positive, see J. Janszky and T. Kobayashi, Phys. Rev. A **41**, 4074 (1990) for a study of Fock states and Ref. [11] for the general case.
 - [8] Other experimental aspects such as a finite number of reference phases and a finite quadrature resolution are analyzed in U. Leonhardt and M. Munroe, Phys. Rev. A **54**, 3682 (1996) and in Ref. [10]. See also Section 5.3 of

